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# Avionics/Control co-design for large flexible space structures

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In this paper, a multi-model  $H_\infty$  synthesis scheme for fixed-structure controller design is developed and applied to the attitude control of a highly flexible earth-observation satellite. The novelty of the proposed approach is that the decision variables optimized by the fixed-structure  $H_\infty$  solver include the structured controller parameters but also some parameters which characterize the avionics. Furthermore the proposed control scheme can be very easily adapted to a new configuration of sensors and thus can handle gyro or gyroless configurations. This way, various avionics configurations can be easily evaluated. The avionics characteristics for a given configuration and the control law can be simultaneously optimized avoiding time-consuming iterations between the definition of avionics and the design of the controller on the basis of the current avionics. The approach is applied on an earth observation satellite for two different study cases. The first one aims to design an improved controller in order to meet the nominal requirements with a poor avionics. The second ones aims to find a controller and an improved avionics to meet very challenging requirements.

## Nomenclature

$MB$	subscript referring to the main body
$A_i$	subscript referring to the $i$ -th appendage
$N$	number of flexible appendages
$P_i$	anchorage point of $i$ -th appendage on main body
$B$	main body center of gravity
$\vec{a}_B$	linear acceleration vector of the main body at point $B$
$\vec{\omega}$	absolute angular velocity vector of the main body
$\vec{\theta}$	absolute angular position of the main body
$\vec{F}_{ext}$	external forces applied to the main body
$\vec{T}_{ext,B}$	torques vector applied to the main body at point $B$
$\tau_{M_i M_j}$	geometric model between points $M_i$ and $M_j$
$\Delta$	matrix of uncertainties
$\Delta_{wc}$	worst case value of $\Delta$
$I_n$	identity matrix of order $n$
$0_{n \times m}$	null $n \times m$ matrix
$J_X, J_Y, J_Z$	diagonal of the total inertia matrix of the spacecraft in the main body axes at point $B$
$T_0$	nominal transmission delay
$X_{ret}$	relative variation of the transmission delay
$n_K$	controller order
$N_{WC}$	number of worst cases considered in the multi-model design
$\mathcal{S}_{n_K}^{p \times m}$	set of stable, $m$ inputs, $p$ outputs minimal $n_K$ -th order linear system

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$\text{diag}(A, B)$	matrix with elements $A$ and $B$ on diagonal, 0 otherwise
$s$	LAPLACE variable
$P(i : j, k : l)$	the sub-system of $P$ between outputs $i$ to $j$ and inputs $k$ to $l$
$F_l(P, K)$	lower LFT of $P$ and $K$
$F_u(P, \Delta)$	upper LFT of $P$ and $\Delta$
<i>Acronym</i>	
LFT	Linear Fractional Transformation
dof	degrees of freedom
AOCS	Attitude and Orbit Control System
SST	Star Sensor
RWA	Reaction Wheel Actuator

## I. Introduction

Space system engineering requires some tools to manage main trade-off as soon as possible in the spacecraft design process. The classical process where the control engineer has to design a control law meeting some specifications for a given mechanical and avionics architecture can bring some time-consuming iterations with the overall system designer. Designing main mechanical parameters, main avionics characteristics and the associated control law in one shot could save lots of time. Today some tools are available to optimize the parameters of a fixed-structure control system and it seems interesting to include in the set decision variables some parameters characterizing the mechanical or avionics design. A cost function must be associated to these parameters to manage the trade-off between the actual cost due to an update of the mechanical/avionics parameters and the classical performance index. With these considerations in mind, the application detailed in this paper concerns the attitude control of a large earth observation satellite where the avionics is parametrized. The same control problem can manage gyro or gyroless configurations in the  $H_\infty$  framework and try to find a controller meeting the 3-axis pointing requirements with the “cheapest” avionics and a decentralized (axis per axis) stable controller. Another methodological interest of such tools concerns the case where the pointing requirement, and so the attitude control design, is very challenging. Then it could be interesting to evaluate solutions with an upgraded avionics.

At this stage of the study the quality of the avionics is characterized by an equivalent transmission delay which is very determinant for this kind of application. Indeed, for the control of a such flexible structure with collocation between actuators and sensors, the transmission delay of the avionics can make the natural positivity of the system to be lost and can raise some parametric robustness problems.<sup>1,2</sup>

Co-design in the field of flexible structure control was already addressed in the literature under the term “integrated design”.<sup>3-6</sup> These references address in fact the integrated design of control gains, sensors and actuators location and also structural optimization. Most of the previous works proposed iterative procedures between mechanical design and control design which can be time consuming for complex mechanical systems. One of the main contribution of the work presented here is to optimize in the same procedure the controller parameters and avionics parameters thanks to recent development on non-smooth optimization<sup>7</sup> and to develop a user-interactive (or user-friendly) design tool such that a new overall closed-loop design can be quickly prototyped.

In the next section of this paper, the modeling of an uncertain flexible spacecraft taking into account the sub-structures of the vehicle are recalled.<sup>8,9</sup> Requirements for the study case are presented in section III. Section IV details the control design methodology. It is based on a fixed-structure  $H_\infty$  design on a multi-channel control problem where the performance in terms of disturbance rejection is handled through a channel weighting the acceleration sensitivity function<sup>10,11</sup> while strong stabilization and roll-off requirements are handled through channels weighting (axis by axis) the open-loop controller to be optimized.<sup>12</sup> Robust stability to face parametric uncertainties is indirectly solved using a multi-model approach to take into account worst-case models selected by  $\mu$ -analysis. This section ends by numerical results on two study-cases: the nominal one and a more challenging study-case. The co-design approach and its application to the two study-cases are detailed in section V. Section VI concludes and presents some perspectives.

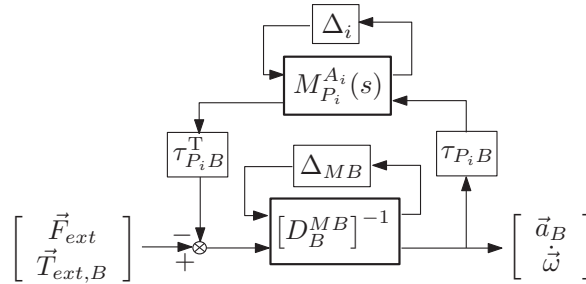
## II. Model

### II.A. Mechanical model

Spacecraft are very complex mechanical multi-body systems including flexible and/or rotating appendages. The design of the AOCS requires a linear model (only valid for low magnitude motions) taking into account all the rigid and flexible couplings between the main body ( $MB$ ) fitted with the avionics hardware of the AOCS (collocated actuators and sensors) and the various appendages  $A_i$  ( $i = 1, \dots, N$ ). The idea of the papers<sup>8,13</sup> is to introduce a multi-body modeling approach which splits the dynamic model of each body within the global system, before connecting them. The dynamic couplings between the main body  $MB$  and a flexible appendage  $A_i$  can be represented by the block diagram depicted in Figure 1 where the direct dynamic model of the appendage  $M_{P_i}^{A_i}(s)$  (including its flexible modes) acts as a feedback on the inverse dynamic model  $[D_B^{MB}]^{-1}$  of the main body. The kinematics model  $\tau_{P_i B}$  depends only on the geometry between the frame attached to the appendage at the anchorage point  $P_i$  on the main body and the main body frame at point  $B$ . The inverse dynamic model of the whole spacecraft  $[M_B^{MB+A_i}]^{-1}$  between the 6 components external forces/torques and 6 components linear/angular accelerations at point  $B$  is thus expressed in the main body axes.

This approach has the advantages

- to fit into the block-diagram as many (rigid or flexible) appendages as possible through other feedbacks on  $[D_B^{MB}]^{-1}$ ,
- to repeat a minimal number of occurrences the physical parameters of each body, that is: mass and inertia for the main body in the model  $D_B^{MB}$  and the flexible mode frequencies, damping ratios and modal participation factors of the cantilevered (at point  $P_i$ ) flexible appendage in the model  $M_{P_i}^{A_i}(s)$ ,
- to directly access to these physical parameters and finally to take easily into account their uncertainties  $\Delta_{MB}$  for the main body parameters and  $\Delta_i$  for the  $i$ -th appendage parameters. This approach guarantees that the global LFT representation of the whole system is minimal and that non-physical parametric configurations for any values of the uncertainties  $\Delta_i$  and  $\Delta_{MB}$  are avoided.



**Figure 1.** Inverse dynamic model  $[M_B^{MB+A_i}]^{-1}$  of the spacecraft taking into account uncertainties  $\Delta_{MB}$  on main body  $MB$  and  $\Delta_i$  on appendage  $A_i$ .

The way to build the dynamic model  $M_{P_i}^{A_i}(s)$  of the flexible appendage directly from the rough data (provided by finite element software) expressed in terms of flexible mode frequencies, damping ratios and modal participation factors is described in.<sup>12</sup>

Numerical applications: the following numerical results consider a low orbit earth observation satellite which is composed of a main body and 2 flexible appendages ( $N = 2$ ): a flexible solar panel (appendage # 1 including 12 flexible modes) and a large deployable antenna (appendage # 2 including 22 flexible modes). The size of the uncertainty block  $\Delta = \text{diag}(\Delta_{MB}, \Delta_1, \Delta_2)$  is  $92 \times 92$ .  $\Delta$  includes uncertainties (10 % relative variations) on main body mass and inertia, pulsations and main modal participation factors on the first 4 flexible modes of each appendage. The nominal ( $\Delta = 0$ ) frequency-domain response (singular values) of the 68-th ( $2 \times (12 + 22)$ ) order model  $[M_B^{MB+A_1+A_2}]^{-1}$  restricted to the 3 attitude dofs is depicted in Figure 2. The total static inertias on the 3 axes (DC gain) are:

$$J_X = 9000 \text{ Kg}m^2, J_Y = 3000 \text{ Kg}m^2, J_Z = 9000 \text{ Kg}m^2.$$

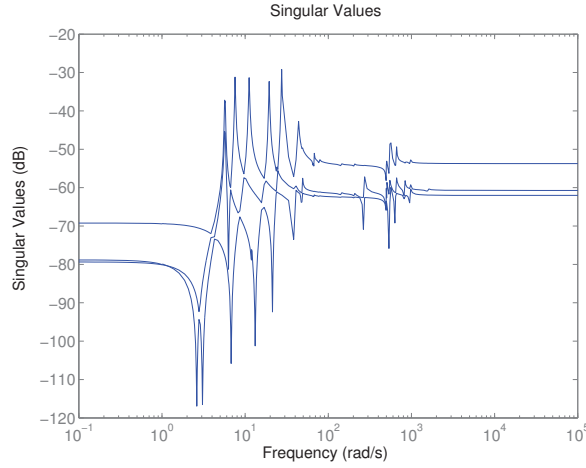


Figure 2. Nominal ( $\Delta = 0$ ) frequency-domain response of  $[M_B^{MB+A_1+A_2}]^{-1}(4:6, 4:6)$ .

## II.B. Avionics

The previous dynamics model is then restricted to the 3 attitude dofs and augmented with the double integrations between accelerations and positions (under small angle assumptions) and the model of the avionics that is:

- the star sensor (SST) to measure the spacecraft attitude  $\vec{\theta}$  which is modelled by a first order transfer with a bandwidth of  $10\text{ Hz}$  on the 3 axis:

$$\text{SST} = \frac{20\pi}{s + 20\pi} I_3 ,$$

- the gyrometers (GYRO) to measure the spacecraft angular rate  $\vec{\omega}$  which are modelled by a first order transfer with a bandwidth of  $200\text{ Hz}$  on the 3 axis:

$$\text{GYRO} = \frac{400\pi}{s + 400\pi} I_3 ,$$

- the reaction wheel actuators (RWA) to actuate torques on the main. RWA are modelled by a first order transfer with a bandwidth of  $100\text{ Hz}$  on the 3 axis. In addition, the transmission delay  $T$  of the overall avionics is modelled by a second order PADE filter  $R(T, s)$  on the 3 axis. The nominal value  $T_0$  of the delay is  $T_0 = 120\text{ ms}$ :

$$\text{RWA} = \frac{200\pi}{s + 200\pi} R(T_0, s) I_3 \quad \text{with} \quad R(T, s) = \frac{s^2 - \frac{6}{T}s + \frac{12}{T^2}}{s^2 + \frac{6}{T}s + \frac{12}{T^2}} .$$

The model  $G(s, \Delta)$  considered for attitude control design is then depicted in Figure 3 where  $M_B^{-1}(s, \Delta) = [M_B^{MB+A_1+A_2}]^{-1}$  for a given value of the uncertainty  $\Delta$ .

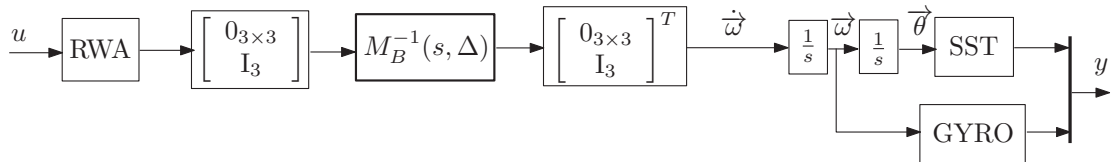


Figure 3.  $G(s, \Delta)$ : attitude dynamic model taking into account avionics.

### III. Requirements

Pointing performances are expressed on the main body. The objective is to reject low frequency orbital disturbances (gravity gradient,...). These disturbances are expressed as a worst-case constant torque  $T_i$  on each axis  $i = X, Y, Z$  to be rejected according to the maximal steady-state pointing errors  $\Delta\theta_i$  summarized in Table 2.

$i$	$X$	$Y$	$Z$
$T_i^{pert} (Nm)$	$1.4 \cdot 10^{-2}$	$2 \cdot 10^{-4}$	$3.9 \cdot 10^{-3}$
$\Delta\theta_i (deg)$	0.03	0.06	0.06

Table 2. Pointing requirements and disturbing torques on  $X$ ,  $Y$  and  $Z$  axes.

Furthermore, a **stable** and **decentralized** controller is recommended (to address easily each axis requirements):

$$K(s) = \text{diag}(K_X(s), K_Y(s), K_Z(s))$$

with the following stability margins on the nominal open loop transfer function  $K(s)G(s, 0)$ :

- gain margin  $> 6 \text{ dB}$  on each axis,
- phase margin  $> 30 \text{ deg}$  on each axis,
- modulus margin  $> 0.5$ .

Finally, the closed-loop system must be stable for any values of the structured parametric uncertainty  $\Delta$  (normalized between  $-1$  and  $1$ ).

### IV. Control design

#### IV.A. Methodology description

From the pointing requirement, the torque disturbance and assuming the spacecraft is rigid, one can easily compute the attitude servo-loop bandwidth required on each axis:

$$\omega_{des,i} = \sqrt{\gamma_{obj} \frac{T_i^{pert}}{J_i \Delta\theta_i}}, \quad i = X, Y, Z$$

$\gamma_{obj}$  is a performance margin to prevent side-effect in the trade-off with others specifications (mainly the roll-off specification). Typically  $\gamma_{obj} = 1.5$ .

Considering that sensors and actuators are collocated on the main body, the transfer between  $u$  (torques applied by AOCS) and  $\omega$  (angular rates) is positive and a classical proportional derivative (PD) control law on each axis:

$$u_i = -K_{p_i} \theta_i - K_{v_i} \omega_i, \quad i = X, Y, Z$$

is guaranteed to stabilize all the flexible modes of the spacecraft. The gains  $K_{p_i}$  and  $K_{v_i}$  can be tuned to meet the required bandwidth  $\omega_{des,i}$  with a good damping ratio  $\xi = 0.7$ :

$$K_{p_i} = J_i \omega_{des,i}^2, \quad K_{v_i} = 2\xi J_i \omega_{des,i}, \quad i = X, Y, Z. \quad (1)$$

When such a control law is applied to the full-order model  $G(s, \Delta)$  including flexible modes and avionics, two situations may occur and can degrade the performance and/or stability of the closed-loop system:

- the required bandwidth is close or beyond the first cantilevered frequency, then this frequency limits the reachable bandwidth by a PD control and the dynamic couplings between flexible and rigid modes make the tuning proposed in (1) is no more valid. This situation is addressed in.<sup>13</sup> It can be easily checked that, for the considered nominal requirements, this is not the case since:

$$\omega_{des,i} \ll \text{1-st cantilevered frequency}, \quad \forall i,$$

- the phase lag due to the avionics can lead the natural positivity of the system to be lost and some flexible modes can be destabilized. In other words, sensors and actuators are collocated from the space-position point of view but from the time-position point of view.

Regarding this last situation, the transmission delay in this application is quite large and may raise some problems. It requires to augment the classical PD law by a phase lead. Such a phase lead is always antagonist with the roll-off commonly specified on the frequency-domain response of the controller to reject sensor noises and/or attenuate the control signal magnitude around high frequency neglected dynamics. Note that from a methodological point of view, its could be very interesting to design a low order controller (corresponding to the PD control augmented with a phase lead) directly on the full order model  $G(s, \Delta)$  taking into account all the flexible modes and the avionics.

With these considerations in mind, the attitude control design approach is based on a fixed-structure  $H_\infty$  design weighting the acceleration sensitivity function.<sup>12</sup> The  $H_\infty$  standard problem  $P(s, \Delta)$  to cope with the pointing requirement is then depicted in Figure 4. The main interests of this scheme are :

- the weighting function  $W_{perf}$  is directly linked to frequency domain requirement  $\omega_{des,i}$ ,
- the weighting function  $W_{perf}$  is independent of the used measurement and does not need to be updated to evaluate gyro or gyroless configurations (Fig. 4 corresponds to the gyro case),
- the actuator dynamics RWA and the sensors dynamics SST and GYRO can be directly taken into the synthesis scheme.

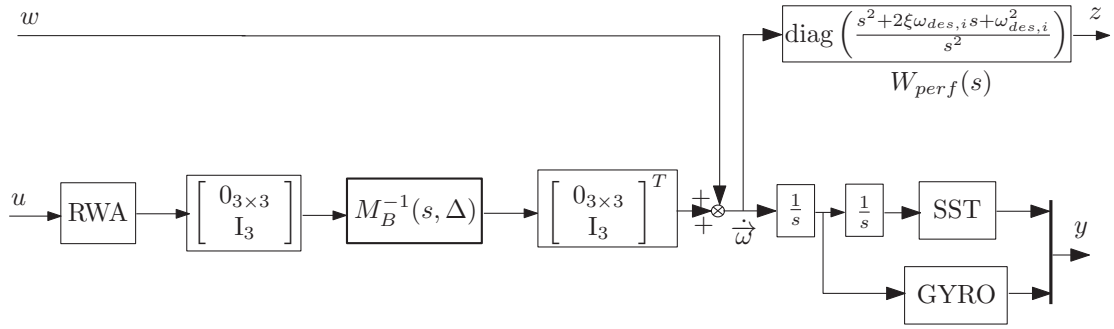


Figure 4.  $P(s, \Delta)$ :  $H_\infty$  standard problem based on the acceleration sensitivity function.

Remark: the standard problem  $P(s, \Delta)$  exhibits a direct feed-through equal to  $I_3$  between  $w$  and  $z$  for all specifications  $\omega_{des,i}$ , thus  $\|F_l(P(s, \Delta), K(s))\|_\infty \geq 1$  for any stabilizing controller  $K(s)$ . That is the reason of the introduction of the performance margin  $\gamma_{obj}$ . Indeed, the pointing requirement is satisfied if:

$$\|F_l(P(s, \Delta), K(s))\|_\infty < \gamma_{obj}.$$

Fixed-structure  $H_\infty$  syntheses<sup>14</sup> allows a low-order structured (here decentralized) controller to be designed on such a problem. The strong stabilization and the roll-off requirements can be directly handled through a weight  $\frac{1}{W_{u_i}(s)}$  on each controller  $K_i(s)$  and the  $H_\infty$  constraint:

$$\left\| \frac{1}{W_{u_i}(s)} K_i(s) \right\|_\infty < \gamma_{obj} \quad i = X, Y, Z.$$

$W_{u_i}(s)$  is the template to be met by the frequency-domain response of the controller  $K_i(s)$ . For numerical application, a  $-20 \text{ dB/dec}$  roll-off is required beyond the pulsation  $\omega_{cut}$  and over three decades (on each axis). The templates  $W_{u_i}(s)$ ,  $i = X, Y, Z$  are represented in Figure 10 (solid green lines). They are defined as a pseudo integral filter (in order to be invertible) and a gain depending on the values of  $\omega_{des,i}$ ,  $J_i$  (in order to the design can work without constraints on the controller low frequency response) and  $\omega_{cut}$ :

$$W_{u_i} = \frac{2}{\gamma_{obj}} J_i \omega_{des,i} \omega_{cut} \frac{1 + \frac{1}{1000\omega_{cut}} s}{s}.$$

Note that all  $H_\infty$  constraints are normalized with respect to the prescribed value  $\gamma_{obj} > 1$ . The last tuning parameter is the order of each controller  $K_i(s)$ ,  $i = X, Y, Z$ . It is recommended to specify a low order in the  $H_\infty$  design process (at least the order of the roll-off filter  $W_{u_i}(s)$  in the gyro case) and to increase the order  $n_K$  if the value of the performance is too far from the objective  $\gamma_{obj}$ . In this application, the controller order  $n_K$  is the same for the three axes. Let us define by  $\mathcal{S}_{n_K}^{p \times m}$  the set of stable,  $m$  inputs,  $p$  outputs,  $n_K$ -th order minimal linear systems. Considering the nominal model ( $\Delta = 0$ ), the controller design is defined by the following procedure:

**Procedure IV.1** *Nominal design procedure:*

**step 1**  $n_K = 1$

**step 2** compute three stabilizing controllers  $\widehat{K}_X(s)$ ,  $\widehat{K}_Y(s)$ ,  $\widehat{K}_Z(s)$  such that:

$$\{\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s)\} = \arg \min_{K_X, K_Y, K_Z \in \mathcal{S}_{n_K}^{1 \times 2}} \max(\dots \quad (2)$$

$$\begin{aligned} & \|F_l(P(s, 0), \text{diag}(K_X(s), K_Y(s), K_Z(s)))\|_\infty, \dots \\ & \|1/W_{u_X} K_X(s)\|_\infty, \dots \\ & \|1/W_{u_Y} K_Y(s)\|_\infty, \dots \\ & \|1/W_{u_Z} K_Z(s)\|_\infty \end{aligned}$$

$$= \arg \min_{K_X, K_Y, K_Z \in \mathcal{S}_{n_K}^{1 \times 2}} \gamma_0(K_X(s), K_Y(s), K_Z(s)). \quad (3)$$

**step 3**  $\widehat{K}(s) = \text{diag}(\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s))$ ,

- if  $\widehat{\gamma}_0 = \gamma_0(\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s)) < \gamma_{obj}$ ,
- then the design is ended,
- else  $n_K = n_K + 1$  and goto step 2,
- endif

Note that convergence of such a procedure is not guaranteed but it can be checked that the distance to the objective  $\widehat{\gamma}_0 - \gamma_{obj}$  decreases when the number of decision variables (directly linked to the order  $n_K$ ) increases (see Table 3).

The stability robustness against parametric uncertainties  $\Delta$  is indirectly solved using an iterative multi-model approach.<sup>12</sup> The  $\mu$  lower bound ( $\underline{\mu}$ ) of  $F_l(P(s, \Delta), \widehat{K}(s))$  is then computed:<sup>15,16</sup>

if  $\underline{\mu}_\Delta(F_l(P(s, \Delta), \widehat{K}(s))) > 1$  then the corresponding worst-case parametric configuration  $\Delta_{wc}$  is taken into account using the following multi-model design (iterative) procedure:

**Procedure IV.2** *Multi-model iterative design procedure:*

**step 0**  $N_{WC} = 0$ . Execute the 3 steps of procedure IV.1.

- step 4**
- if  $\underline{\mu}_\Delta(F_l(P(s, \Delta), \widehat{K}(s))) < 1$
  - then the design is ended,
  - else collect the worst case uncertainty  $\Delta_{WC}$ ,  $N_{WC} = N_{WC} + 1$
  - endif

**step 5** compute three stabilizing controllers  $\widehat{K}_X(s)$ ,  $\widehat{K}_Y(s)$ ,  $\widehat{K}_Z(s)$  such that:

$$\{\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s)\} = \arg \min_{K_X, K_Y, K_Z \in \mathcal{S}_{n_K}^{1 \times 2}} \max(\dots \quad (4)$$

$$\begin{aligned} & \gamma_{N_{WC}-1}(K_X(s), K_Y(s), K_Z(s)), \dots \\ & \|F_l(P(s, \Delta_{WC}), \text{diag}(K_X(s), K_Y(s), K_Z(s)))\|_\infty \end{aligned}$$

$$= \arg \min_{K_X, K_Y, K_Z} \gamma_{N_{WC}}(K_X(s), K_Y(s), K_Z(s)), \quad (5)$$



**step 6**  $\widehat{K}(s) = diag(\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s)),$

- if  $\widehat{\gamma_{N_{WC}}} = \gamma_{N_{WC}}(\bar{K}_X(s), \bar{K}_Y(s), \bar{K}_Z(s)) > \gamma_{obj}$ ,
- then  $n_K = n_K + 1$  and goto step 5,
- else if  $\underline{\mu}_{\Delta}(F_l(P(s, \Delta), \hat{K}(s))) < 1$ ,
  - then the design is ended,
  - else collect the worst case uncertainties  $\Delta_{WC}$ ,  $N_{WC} = N_{WC+1}$  and goto step 5,
  - endif
- endif

At the end of the procedure  $N_{WC}$  is the number of worst-case models taken into account in the multi-model synthesis in addition to the nominal model. Note also that convergence of such a procedure is not guaranteed and it is also recommended to end the procedure once  $N_{WC}$  is greater than a prescribed value (typically: 5).

Note also that at the end of procedure, the  $\mu$  upper bound must be computed to ensure the robust stability, parametrically.

#### IV.B. Results on the nominal study-case

The nominal study case corresponds to the specifications described in Table 2. The required bandwidth (with  $\gamma_{obj} = 1.5$ ) are:

$$\omega_{des,X} = 0.07 \text{ (rad/s)}, \omega_{des,Y} = 0.01 \text{ (rad/s)}, \omega_{des,Z} = 0.03 \text{ (rad/s)},$$

and there are quite under the first cantilevered pulsation (around  $2.5 \text{ (rad/s)}$ , see anti-resonances in Figure 2). The roll-off frequency is  $\omega_{cut} = 1 \text{ (rad/s)}$  on each axis.

In fact these specifications are not very challenging and the procedure IV.2 provides the following results:

$$n_K = 2, \quad N_{WC} = 0, \quad \hat{\gamma}_0 = 1.16, \quad \underline{\mu}_\Delta(F_l(P(s, \Delta), \hat{K}(s))) = 0.1.$$

That is: a very efficient and robust solution with a 2-nd order controller per axis. The corresponding NICHOLS responses are presented in Figure 5. Stability margins are quite comfortable. This first decentralized controller is denoted  $K_1(s)_{3 \times 6}$  and its frequency-domain responses (for the 3 axes) are plotted in Figure 10 (dashed green lines).

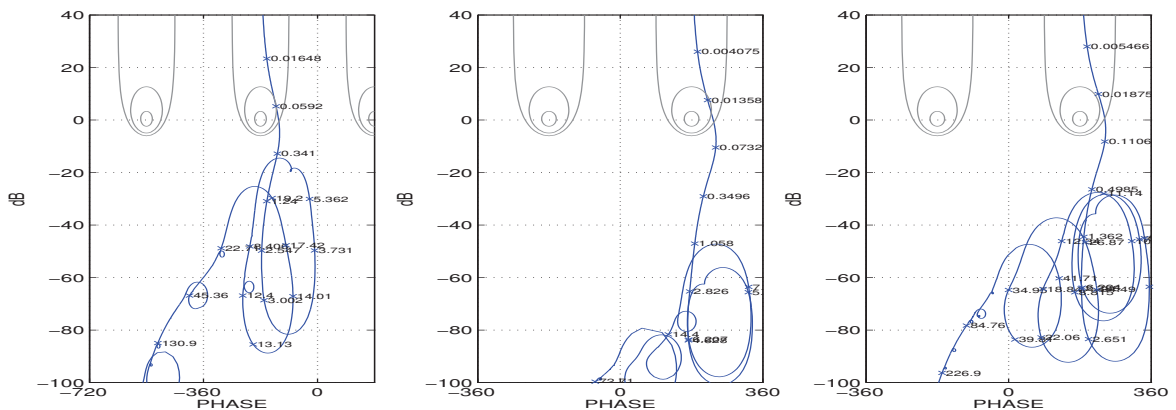


Figure 5. Nominal Nichols plot for the nominal avionics and the controller  $K_1(s)$ :  $X$  (left),  $Y$  (center),  $Z$  (left) axes.

These results lead to the following question:

*Is it possible to have the same performance (or at least:  $\widehat{\gamma_{NWC}} < \gamma_{obj} = 1.5$ ) and the same parametric robustness with a degraded (low-cost) avionics, eventually with a gyroless configuration and at a price of a more complex controller?*

The section V aims to give some answers.

#### IV.C. Results on the challenging study-case

In order to assess the overall methodology, the control design approach is now evaluated on a more challenging study-case. This challenging study case considers only the  $X$  axis of the spacecraft but the required bandwidth is increased up to  $\omega_{des,X} = 1 \text{ rd/s}$ . Consequently, the roll-off pulsation is set to  $\omega_{cut_X} = 10 \text{ rd/s}$ . The required bandwidth is now very close to the first cantilevered frequency and the controller have to cope with dynamics couplings between rigid and flexible modes. In the frequency-domain, that means there is not a gap wide enough to attenuate the flexible modes using a (stable) low-pass filter and the flexible modes must be now phase-controlled. The tuning of such a controller becomes tricky when the phase lag due to the avionics (mainly the transmission delay) and parametric uncertainties are taken into consideration.

Table 3 gives the influence of the controller order  $n_K$  on the index performance  $\hat{\gamma}_0$  applying the nominal design procedure IV.1 (without any parametric robustness constraints) and highlights that increasing the order of controller (that is the number of decision variables in the optimization process) allows to improve the performance index:  $\hat{\gamma}_0 < \gamma_{obj}$  ( $= 1.5$ ) for  $n_K = 5$ .

$K_X(s)$ order ( $n_K$ )	2	3	4	5	6
$\hat{\gamma}_0$	4.54	2.89	1.66	1.45	1.26

Table 3. Evolution of  $\hat{\gamma}_0$  according to the controller order  $n_K$  during the nominal design procedure IV.1.

Applying the multi-model iterative procedure IV.2 allows to find a first worst-case parametric configuration which destabilizes the closed-loop system (see Figure 6). Taking into account iteratively the selected worst-cases allows the parametric robustness to be improved (that is:  $\mu_\Delta$  to be decreased), but after 5 worst-case models, there are still some robustness problems. This result leads to the following question:

*Is it possible to meet robustness and challenging requirements with an upgraded avionics ?*

The section V aims to give some answers.

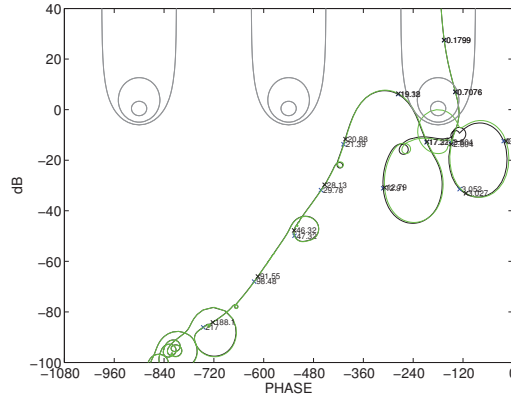


Figure 6. Nichols plots on the nominal model (black: stable), on the worst case model # 1 (green: unstable).

#### V. Avionics and control co-design

The objective is to optimize simultaneously (and not iteratively) the controller parameters and the avionics represented by some tunable parameters. In this study, it is roughly assumed that the quality of the avionics can be represented by the transmission delay  $T$  and that this transmission delay can be adjusted around a nominal value  $T_{nom}$ . That is:

$$T = T_{nom}(1 + X_{ret}) \quad (T_{nom} = 120 \text{ ms}),$$

where  $X_{ret} > -1$  is a relative variation to be optimized.

Fixed-structure  $H_\infty$  design can also be used to optimize in the same procedure  $X_{ret}$  and the controller. It is thus possible to find the maximal admissible transmission delay  $T_{max}$  and the associated controller meeting the nominal specifications.

The second order *Pade* approximation  $R(T, s)$  of a varying delay  $T = T_{nom}(1 + X_{ret})$  can be represented by an LFT:

$$R(T, s) = F_u(D(T_{nom}, s), X_{ret}I_2)$$

where  $D(T_{nom}, s)$  is depicted in Figure 7.

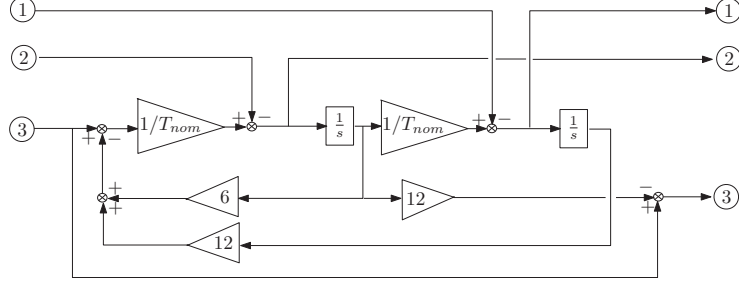


Figure 7. LFT  $D(T_{nom}, s)$  of a tunable delay based on a second order Pade approximation.

Such an LFT must be taken into account on the 3 axes and the basic standard problem  $P(s, \Delta)$  is changed to the standard problem  $P_T(s, \Delta)$  presented in Figure 8 and where 6 additional control signals  $u_a$  and 6 additional measurements  $y_a$  appear to feedback the relative variation of the delay through the gain  $X_{ret}I_6$ .

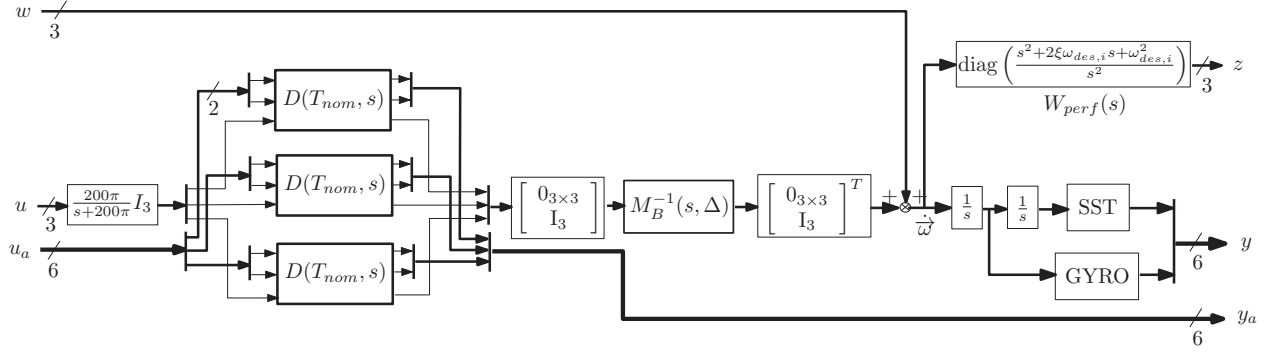


Figure 8.  $P_T(s, \Delta)$ :  $H_\infty$  control standard problem for avionics/control co-design.

The structure of the augmented controller to be optimized on this standard problem reads:

$$\mathbf{K}_{9 \times 12} = \begin{bmatrix} \begin{bmatrix} K_X(s)_{1 \times 2} & & \\ & K_Y(s)_{1 \times 2} & \\ & & K_Z(s)_{1 \times 2} \end{bmatrix} & \phi_{3 \times 6} \\ \begin{bmatrix} & & \\ & \phi_{6 \times 6} & \\ & & X_{ret}I_6 \end{bmatrix} \end{bmatrix} = \text{diag}(K_X(s), K_Y(s), K_Z(s), X_{ret}I_6).$$

The problem is now to find three stabilizing controllers  $\widehat{K}_X(s)$ ,  $\widehat{K}_Y(s)$ ,  $\widehat{K}_Z(s)$  and a delay relative variation  $\widehat{X}_{ret}$  such that:

$$\{\widehat{K}_X(s), \widehat{K}_Y(s), \widehat{K}_Z(s), \widehat{X}_{ret}\} = \arg \min_{K_X, K_Y, K_Z \in \mathcal{S}_{n_K}^{1 \times 2}, X_{ret} > -1} \max(\gamma_{N_{WC}}, f(X_{ret})) \quad (6)$$

where

- $\gamma_{N_{WC}} = \gamma_{N_{WC}}(K_X(s), K_Y(s), K_Z(s), X_{ret})$  is defined,  $\forall N_{WC} \geq 0$ , by equations (2) to (5) changing  $F_l(P(s, \dots), \text{diag}(K_X(s), K_Y(s), K_Z(s)))$  by  $F_l(P_T(s, \dots), \text{diag}(K_X(s), K_Y(s), K_Z(s), X_{ret}I_6))$ ,
- $f(X_{ret})$  is a (decreasing) penalty function and must be chosen in order to maximize  $X_{ret}$  in the balance of the new index:  $\max(\gamma_{N_{WC}}, f(X_{ret}))$ , considering that at the optimum (minimum) the 2 components of the max function are balanced:  $\min \max(\gamma_{N_{WC}}, f(X_{ret})) = \widehat{\gamma}_{N_{WC}} = f(\widehat{X}_{ret})$ .

The avionics-control co-design mainly consists to change equations (3) and (5) by equation (6) in steps 2 and 5 of procedures IV.1 and IV.2. Finally, the last methodological problem is the choice of the penalty function  $f(X_{ret})$ . The following function:

$$f(X_{ret}) = \frac{K_{ret}}{X_{ret} + 1} \quad \text{with: } K_{ret} > 0$$

will be considered regarding the following properties:

- $f(X_{ret}) > 0, \quad \forall X_{ret} > -1,$
- $f(-1) = \infty$ : a null transmission delay is highly penalized,
- $\lim_{X_{ret} \rightarrow \infty} f(X_{ret}) = 0$ : a big delay is encouraged,
- $K_{ret} = f(0)$  is the penalty function value for the nominal delay  $T_{nom}$ .  $K_{ret}$  must be adjusted according to the value of  $\widehat{\gamma_{NWC}}$  (obtained with the nominal avionics) and the following considerations:
  - if the control problem with the nominal avionics is challenging, i.e.:  $\widehat{\gamma_{NWC}} \gg 1$  even with high order controllers or the control design procedure fails to find a parametrically robust solution (this is the case of the application presented in section IV.C), then one can choose  $K_{ret} \approx 1$  to seek for a solution with an upgraded avionics ( $X_{ret} \approx \frac{1}{\gamma_{NWC}} - 1 < 0$ ),
  - if the control problem with the nominal avionics is not challenging, i.e.:  $\widehat{\gamma_{NWC}} \approx 1$  even with low order controllers (this is the case of the application presented in section IV.B), then  $K_{ret}$  can be increased to find a solution with a degraded avionics (for instance  $K_{ret} = 10$ ).

### V.A. Co-design on the nominal study case

The results presented in section IV.B for the nominal specifications and the nominal avionics highlight that the control problem is not challenging. Thus, there are some margins to obtain the same performance/robustness with a degraded avionics (i.e. with a greater transmission delay  $T$ ). For this reason,  $K_{ret} = 10$  was adopted in the penalty function  $f(X_{ret})$ . The co-design leads to the following results:

$$n_K = 2, \quad N_{WC} = 0, \quad \widehat{\gamma}_0 = 1.17, \quad \underline{\mu}_\Delta(F_l(P_T(s, \Delta), \text{diag}(\widehat{K}(s), \widehat{X}_{ret} I_6))) = 0.31, \quad \widehat{X}_{ret} = 7.55 (= \frac{K_{ret}}{\widehat{\gamma}_0} - 1).$$

That is: a very efficient and robust solution with a 2-nd order controller per axis and a very degraded avionics with an important transmission delay ( $T = T_{nom}(1 + X_{ret}) = 1026\text{ ms}$ ). The corresponding NICHOLS responses (taking into account the new transmission delay  $T$ ) are presented in Figure 9. Stability margins are comfortable. This second decentralized controller is denoted  $K_2(s)_{3 \times 6}$  and its frequency-domain responses (for the 3 axes) are plotted in Figure 10 (dashed black lines).

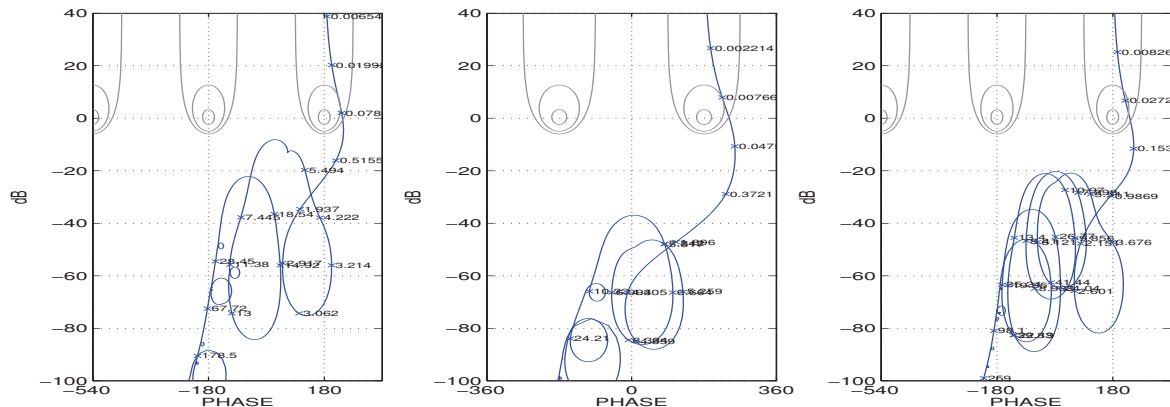


Figure 9. Nichols plots on the model with  $T = 1026\text{ ms}$  and the controller  $K_2(s)$ :  $X$  (left),  $Y$  (center),  $Z$  (left) axes.

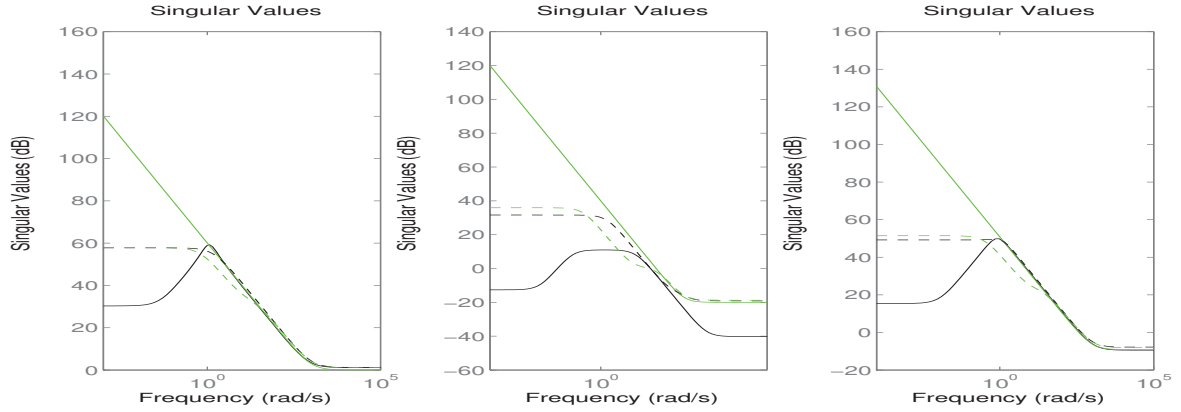


Figure 10. Magnitude plots of controller  $K_1(s)$  (dashed green),  $K_2(s)$  (dashed black),  $K_3(s)$  (solid black) - template  $W_{u_i}(s)$  (solid green): X (left), Y (center), Z (left) axes.

### V.B. Co-design on the nominal gyroless study case

The main interest of the proposed control design approach is that the weighting system, to handle performance and roll-off requirements, that is: the weight  $W_{perf}(s)$  and  $W_{u_i}(s)$ , is directly linked to frequency-domain specifications  $\omega_{des,i}$  and  $\omega_{cut}$  and does not depend on the sensor configuration. Thus, the adaptation of the previous co-design approach can be very easily updated to handle the gyroless configuration and is reduced to the following points:

- remove the outputs # 7, 8 and 9 of the problem  $P_T(s, \Delta)$  (since the measurement of the 3 components of  $\vec{\omega}$  is no more available, see Figure 8),
- change the structure of controller since  $K_X(s)$ ,  $K_Y(s)$  and  $K_Z(s)$  are now single-input single-output transfer functions (that is: change  $\mathcal{S}_{n_K}^{1 \times 2}$  by  $\mathcal{S}_{n_K}^{1 \times 1}$  in equation (6)),
- start the procedure IV.1 (step 1) with  $n_K = 2$  since the controller needs to derive the attitude measurement (to estimate the angular rate) and must have a  $-20 \text{ dB/dec}$  roll-off behavior. Indeed the minimal order to meet the specifications is at least 2 (per axis),
- Finally, to take into account that the SST is only available, the nominal value of the transmission delay is set to  $T_{nom} = 300 \text{ ms}$  and the gain on the penalty function  $f(X_{ret})$  is set to  $K_{ret} = 3$  (that is: the ideal index performance  $\gamma = 1$  corresponds to a quite big delay ( $900 \text{ ms}$  or  $X_{ret} = 2$ )).

The co-design leads to the following results:

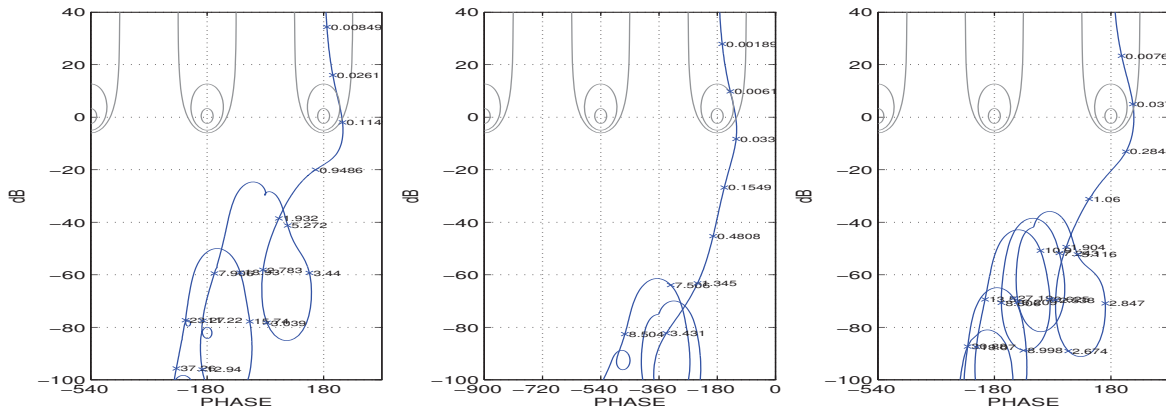
$$n_K = 2, \quad N_{WC} = 0, \quad \hat{\gamma}_0 = 1.17, \quad \underline{\mu}_\Delta(F_l(P_T(s, \Delta), \text{diag}(\hat{K}(s), \hat{X}_{ret} I_6))) = 0.135, \quad \hat{X}_{ret} = 1.56 (= \frac{K_{ret}}{\hat{\gamma}_0} - 1).$$

That is: a very efficient and robust solution with a 2-nd order controller per axis and a degraded gyroless avionics with an important transmission delay ( $T = T_{nom}(1 + X_{ret}) = 770 \text{ ms}$ ). The corresponding NICHOLS responses (taking into account the new transmission delay  $T$ ) are presented in Figure 11. Stability margins are comfortable. This third decentralized controller is denoted  $K_3(s)_{3 \times 3}$  and its frequency-domain responses (for the 3 axes) are plotted in Figure 10 (solid black lines). Although the derivative action of each controller is quite obvious, the templates  $W_{u_i}$  are still satisfied.

### V.C. Co-design on the challenging study case

The co-design on the challenging study case (X-axis) with  $K_{ret} = 1$  and  $T_{nom} = 120 \text{ ms}$  leads to the following results:

$$n_K = 4, \quad N_{WC} = 1, \quad \hat{\gamma}_1 = 1.51, \quad \underline{\mu}_\Delta(F_l(P_T(s, \Delta), \text{diag}(\hat{K}(s), \hat{X}_{ret} I_6))) = 0.6, \quad \hat{X}_{ret} = -0.34 (= \frac{K_{ret}}{\hat{\gamma}_1} - 1).$$



That is: while a 5-th order controller design involving 5 worst-case models failed (from the robustness point of view) with the nominal avionics, an upgraded avionics ( $T = T_{nom}(1 + X_{ret}) = 79\text{ ms}$ ) allows to meet all the specifications with a 4-th order controller (design with only one worst-case model). The corresponding NICHOLS response (taking into account the new transmission delay  $T$ ) is presented in Figure 12. Stability margins are quite comfortable. The frequency-domain response of the controller  $K_X(s)$  is plotted in Figure 13 (solid black line) and satisfies the template  $W_{u_X}$  within the value of  $\gamma_{obj} = 1.5\text{ (3.5 dB)}$ .

## VI. Conclusions and Perspectives

From previous works where fixed-structure  $H_\infty$  synthesis was applied to a multi-channel, multi-model control problem weighting the acceleration sensitivity function, this paper proposed some extensions to take into account some avionics tuning parameters in the optimization process. The so-called avionics/control co-design was applied to spacecraft attitude control design and allowed directly the interest of a gyro or gyroless sensor configuration and of an degraded or upgraded avionics to be evaluated.

The quality of the avionics was roughly represented by the transmission delay: further works need to be performed to access others avionics characteristics: noise, bias, .... But the results obtained in this study are quite promising on the capability of fixed-structure  $H_\infty$  design to cope with high order models and multi-channel, multi-model control problem. Another promising perspective is the mechanical/control co-design where some design mechanical parameters (for instance, mass and stiffness of the boom linking the antenna on the main body) could be optimized with the associated attitude controller. Note that to address such a problem, the modeling tool used in this study must be extended to handle arbitrary kinematics chain of

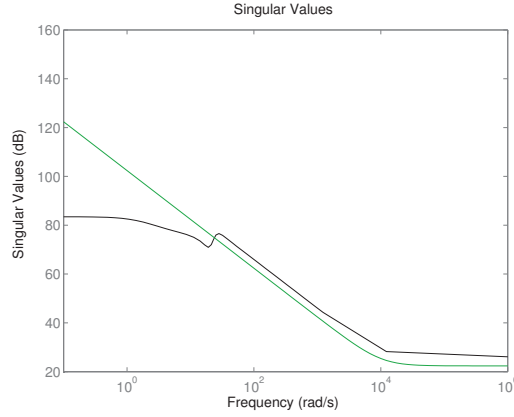


Figure 13. Magnitude plots of controller  $K_X(s)$  (solid black) - template  $W_{uX}(s)$  (solid green).

(tunable) flexible bodies (according to Figure 14 where the appendage  $A_i$  holds the appendage  $A_j$  at its point  $P_{ij}$ ). From the control design methodology, some extensions are also required to cope with pointing requirements on a flexible payload or appendage (antenna for instance) and not only on the main body.

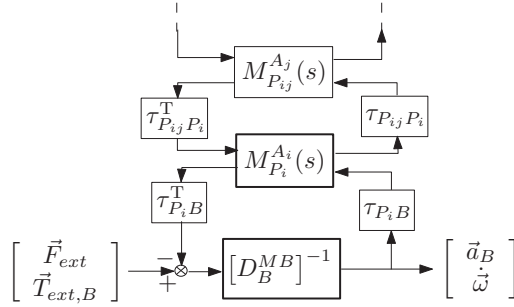


Figure 14. Block diagram representation of a flexible multi-body modeling tool.

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